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A SIMPLE FORMULA TO CALCULATE SHALLOW-WATER TRANSMISSION LOSS BY MEANS OF A LEAST- SQUARES SURFACE FIT TECHNIQUE

bу

OLE F. HASTRUP and TUNCAY AKAL

1 SEPTEMBER 1980

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A SIMPLE FORMULA TO CALCULATE SHALLOW-WATER TRANSMISSION FOR BY MEANS OF A LEAST-SQUARES SURFACE FIT TECHNIQUE,

(10) ole F./Hastrup Tuncay/Akal

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## A SIMPLE FORMULA TO CALCULATE SHALLOW-WATER TRANSMISSION LOSS BY MEANS OF A LEAST-SQUARES SURFACE FIT TECHNIQUE

by

Ole F. Hastrup and Tuncay Akal

#### **ABSTRACT**

A semi-empirical formula  $TL = 15 \log R + [A + B(\log f) + C(\log f)^{\frac{1}{2}}] R + D$  (in dB) is proposed to express sound transmission loss in shallow water as a function of range and frequency. The four coefficients A, B, C and D are determined from either experimental or model data by the use of a least-squares surface fit and the formula usually gives a standard derivation of the order of a few dB. The formula can be used for studying system performances, sonar range predictions, and as a compact data storage.

#### INTRODUCTION

The numerical calculation of transmission loss in shallow water has always been considered with a certain reluctance due to the apparent lack of systematics in the measured results and the rather complicated and lengthy mathematics involved.

Numerous attempts have therefore been made to express the losses by simple empirical or semi-empirical expressions using the knowledge obtained from experiments in model tanks and at sea. Such transmission formulas have especially been adapted for military planning and operational use.

Generally they give transmission losses as function of range for a given fixed frequency, using logarithmic terms to describe the geometrical spreading loss and some additional terms to take the extra boundary and volume attenuation into account. The coefficients in the formulas are usually obtained using experimental results and curve fitting for given discrete frequencies or frequency bands. This means that in order to calculate transmission loss over a wide frequency band a large set of coefficients are needed, thereby reducing the practical value of such formulas.

By recognizing that the general transmission loss is a two-dimensional function of only range and frequency, assuming the pertinent environmental

condition to be invariant during the period of observation, it would be more obvious to use a two-dimensional expression for the transmission loss TL, such as

$$TL = g (f, R, A_1, A_2 ... A_n),$$
 (Eq. 1)

with f being the frequency, R the range, and the coefficients  $A_{\hat{1}}$  determined not by a set of line fits but by a proper surface least-squares fit. In order to keep n as small as possible for a required standard deviation between measured and calculated values, it is extremely important that the chosen formula represents as closely as possible the true physical character of the problem.

#### 1 SOME EXISTING ONE-DIMENSIONAL FORMULAS

The development of semi-empirical one-dimensional transmission loss formulas have been given a significant attention during the last 25 years. One of the earliest works was reported by Brekhovskikh <1>, who calculated the decay of the acoustic intensity averaged over the water column. It shows that for ranges much larger than the water depth the average intensity decreases with distance as  $R^{-3/2}$ , a compromise between the cylindrical law  $R^{-1}$  associated with total reflection from the boundaries and the spherical law  $R^{-2}$  associated with the absence of boundaries, as in deep water for example. At longer ranges, when one assumes the presence of only one mode, the average intensity varies with  $R^{-1} \ e^{-\alpha R}$ , corresponding to cylindrical spreading with an extra exponential damping.

Marsh and Schulkin <2>, using a large number of data, suggested generalized formulas for the loss calculations, dividing the range into short, medium and long and using respectively  $R^{-2}$ ,  $R^{-3/2}$ , and  $R^{-1}$  types of expression with additional terms and coefficients depending on frequency, sea state, and bottom type. The data should be considered with some caution since, for example, they do not predict the important optimum frequency often observed in shallow-water propagation. A simple expression used by several, such as Schelstede and Petersen <3>, gives the transmission loss as:

$$TL = A \log R + B \cdot R + C \quad dB \quad , \tag{Eq. 2}$$

where the coefficients A, B, and C are determined for each frequency band by a least-squares fit.

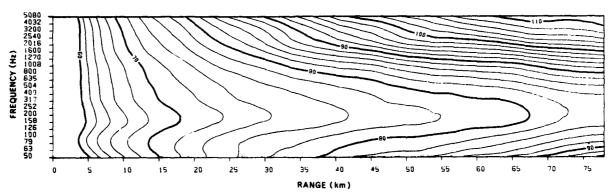
The validity of the above transmission loss laws have been studied in detail by Murphy and Olesen <4>, <5>, <6> using a very large number of experimental data covering frequencies from 100 to 8000 Hz and ranges up to

40 km. The result is that both  $R^{-3/2}$  and  $R^{-1}$   $e^{-\alpha R}$  are usually equal contenders, with  $R^{-3/2}$  being best around the optimum frequency and  $R^{-1}$   $e^{-\alpha R}$  being best when considering the whole frequency range. All the previously mentioned studies have been made only in the range domain and have used not the fact that the loss is equally dependent on frequency. The result is therefore a lack of function fitting in the frequency domain and a creation of a new set of coefficients for each frequency.

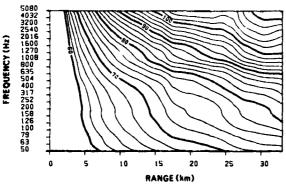
Considering the desirability of having a simple formula with only a few coefficients for each acoustic propagation condition, this clearly leads to an approach using an expression of the type of Eq. 1, with coefficients obtained by two-dimensional, surface, least-squares fits from broadband experimental or predicted data.

## 2 SIMPLE TWO-DIMENSIONAL TRANSMISSION-LOSS DATA

One of the simplest and clearest ways to display functions of two variables is to use iso-contours, which in this case means iso-loss contours in the frequency/ range plane. An inspection of such loss contours from approximate 170 cases examples in shallow water <7> indicates that we are dealing with two characteristic cases, as shown on Figs. 1a and 1b.



(a) marked optimum frequency



(b) no marked optimum frequency

FIG. 1 THE MOST CHARACTERISTIC FAMILIES OF TRANSMISSION LOSS CONTOURS

Figure 1a shows the case where an optimum frequency has been observed within the studied frequency interval; it usually represents propagation over a hard or semi-hard bottom. In Fig. 1b no marked optimum frequency was found in the studied frequency interval; this usually corresponds to propagation over a softer bottom. So the requirement for our general expression, Eq. 1, is that it should contain families of curves similar to the ones in Fig. 1 and include both logarithmic and linear terms of the range R.

After some trial and error the following formulas have emerged:

$$TL = 15 \log R + [A + B (\log f) + C (\log f)^2] R + D$$
, dB, (Eq. 3)

giving the loss contour equation h(f, R) for a fixed loss L

$$h(f, R) = 15 \log R + [A + B(\log f) + C(\log f)^2] R + D - L = 0$$
 (Eq. 4)

Let us have a short look at some of the characteristic features of this family of curves.

Partial differentiation yields:

$$\frac{\partial h}{\partial (\log f)} \approx (B + 2C \cdot \log f) \cdot R$$
 (Eq. 5a)

$$\frac{\partial h}{\partial R}$$
 = 15 log e  $\cdot \frac{1}{R}$  + A + B(log f) + C(log f)<sup>2</sup> (Eq. 5b)

From Eq. 5a we get, for  $\frac{\partial h}{\partial (\log f)} = 0$ ,

$$\log f = -\frac{B}{2C} , \qquad (Eq. \cdot 6)$$

which corresponds to the characteristic optimum frequency (OF) of shallow water.

Using Eq. 5b we get, for  $\frac{\partial h}{\partial R} = 0$  ,

6.51 
$$\frac{1}{R}$$
 + A + B (log f) + C (log f)<sup>2</sup> = 0 , (Eq. 7)

where  $6.51 = 15 \log e$ .

The behaviour of the h (f, R) curves depends on the number of solutions to Eq. 7, which again depends on the following classical condition

$$S = B^2 - C (6.51 \frac{1}{R} + 4A) R = 0$$
.

When S < 0 for all values of R , Eq. 7 has no solutions and  $\frac{\partial h}{\partial R} \neq 0$ ,

which corresponds to the situation of Fig. 1a where the contours have a "parabolic" type of look. The other, for us, important case is where S < 0 for small values of R and S > 0 for large values of R, the latter corresponding to the situation of Fig. 1b where the contours now have a "hyperbolic" type of look. One can define a critical range  $R_{cr}$  that divides between the two cases. From S = 0 we get:

$$R_{cr} = \frac{26.04 \text{ C}}{B^2 - 4 \text{ AC}}$$
.

In other words the point (R f) with the coordinates  $\frac{26.04 \text{ C}}{\text{B}^2-4 \text{ AC}}$ ,  $10^{-\text{B}/2\text{C}}$  is a singular point.

To illustrate this general behaviour, Fig. 2 shows some selected contours for the following values:

$$A = 4.175$$
,  $B = 3.320$ ,  $C = 0.640$  and  $D = 50.0$ ,

demonstrating the two types of solutions with  $R_{\rm cr} \bowtie 50$ . We can therefore conclude that Eq. 4 represents the characteristics of the transmission-loss contours observed in most cases of propagation in shallow water and that Eq. 3:

$$TL = 15 \log R + [A + B(\log f) + C(\log f)^2] R + D$$
 dB

can be considered as a valid, semi-empirical, transmission-loss law for shallow water.

### 3 COEFFICIENT CALCULATIONS

To calculate the four coefficients A, B, C and D, a standard, least-squares, surface-fit technique has been applied. The problem has been programmed for the SACLANTCEN 1106 UNIVAC computer, giving the possibility to use both experimental data and data from models <8>.

As an example, Figs. 3a and 3b show the results using the measured contours seen on Figs. 1a and 1b. It will be noticed that the calculated loss contours closely resemble the measured ones and that the standard deviation is of the order of a few decibels. This figure can be considered to be acceptable when working with real, measured transmission losses. The largest difference between measured and calculated losses occurs for very short ranges. This is partly due to the choice of a 15 log R term, rather than a 20 log R term, (see Ch. 4), and that the measurements were usually initiated at this range. Since we are interested only in the prediction for longer ranges this difference is of no major importance.

To get an idea of the general validity of the formula, the standard deviation has been calculated for approximately 40 measured, depth-averaged, transmission-loss surfaces. This yields a mean of 2.7 dB  $\pm$  1 dB, generally with the highest deviation for upward refraction condition and the lowest for downward or internal-duct conditions.

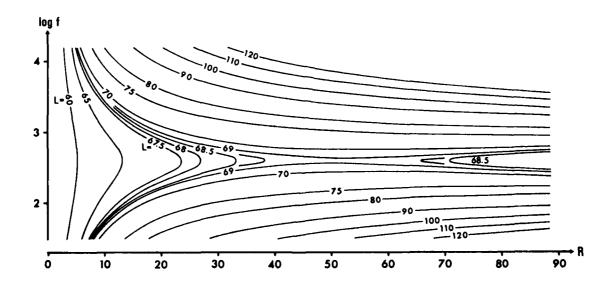
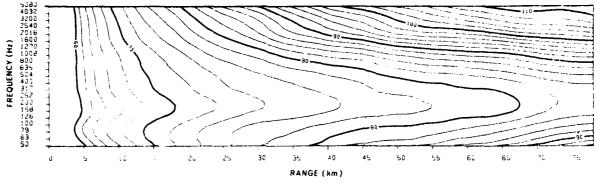
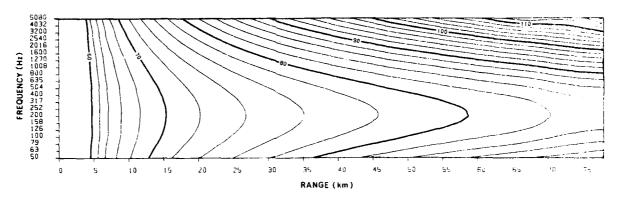


FIG. 2

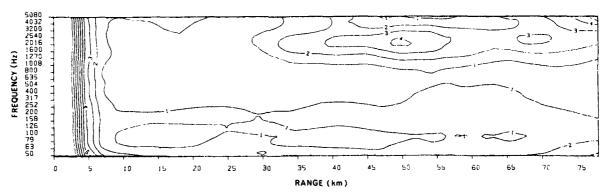
GENERAL CONTOURS FOR  $TL = 15 \log R + (A + B \log f + C (\log f)^2) R + D$ 



Measured loss contours

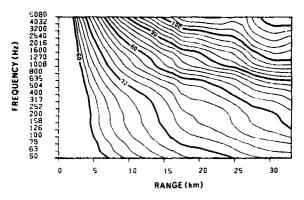


Calculated loss contours

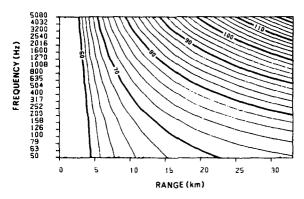


Error loss contours

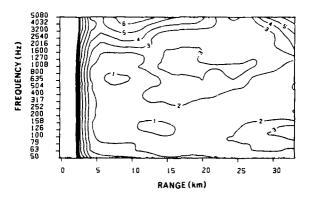
FIG. 3a COMPARISON OF THE MEASURED TRANSMISSION-LOSS CONTOURS OF FIG. 1a WITH CALCULATED TRANSMISSION-LOSS CONTOURS



Measured loss contours



Calculated loss contours



Error loss contours

FIG. 3b COMPARISON OF THE MEASURED TRANSMISSION-LOSS CONTOURS of FIG. 1b WITH CALCULATED TRANSMISSION-LOSS CONTOURS

#### 4 COMMENTS

The choice of 15 as the coefficient for log R was made after testing 10, 12.5, 15, 17.5 and 20 on several acoustic runs. The smallest error was almost equally distributed between 12.5, 15 and 17.5, with 15 being slightly better.

In a few of the experimental cases the optimum frequency varied with range, which means at least one extra coefficient is needed to create this effect. It would not be difficult to include this feature but is is felt that this is not worthwhile since tests show only a small reduction in standard deviation for an extra coefficient.

On some occasions more than one optimum frequency is measured. This is caused by one or more internal sound channels and in order to obtain the best fit to the exprimental data one has to include higher-order terms of log f in Eq. 3, at least up to the fourth degree.

Concerning the use of Eq. 3 to extrapolate results to larger ranges, some caution should be observed. Where we have a marked optimum frequency in the frequency interval, the environmental condition being of course invariant with range, tests show that such extrapolations seem to be possible without serious errors.

But where no marked optimum frequency is observed, and data exist only for a limited range, one has to be very cautious in making significant extrapolations, especially near the optimum frequency. This usually corresponds to the case of S > 0 (Eq. 7) where frequencies exist that satisfy the condition  $\frac{\partial h}{\partial R} = 0$ , meaning that losses could decrease with range.

On the other hand, test calculations have shown that Eq. 3 can be used to extrapolate results to frequencies at least one octave higher, a technique that could be useful where poor signal-to-noise ratio makes high-frequency measurements difficult at long range.

#### CONCLUSIONS

We can therefore conclude that the transmission loss in shallow water can be expressed as a function of both range and frequency by

$$TL = 15 \log R + [A + B(\log f) + C(\log f)^2] R + D$$
 dB

By the use of only four coefficients this formula normally gives results that, inside the frequency and range intervals given by the basic data, are accurate to within a few decibels.

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